TRINITY COLLEGE

Mathematics Applications Unit 3/4
Test 2 2020

Section 1 Calculator Free

Sequences

STUDENT'S NAME SOLUTIONS - WILSON

DATE: Friday 22nd May **TIME:** 25 minutes **MARKS:** 24

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

A sequence is such that its' first 3 terms are 20, 30, 45…

(a) **<u>Show</u>** that the sequence is geometric. [2]

$r = \frac{30}{20} = \frac{45}{30}$ $\therefore$ G.P

✓ shows 1 ratio

✓ shows 2 ratios

(b) Write the recursive rule for the sequence. [2]

$T_{n+1} = \frac{3}{2} T_n, \quad T_1 = 20$

✓ recursive rule

✓ first term

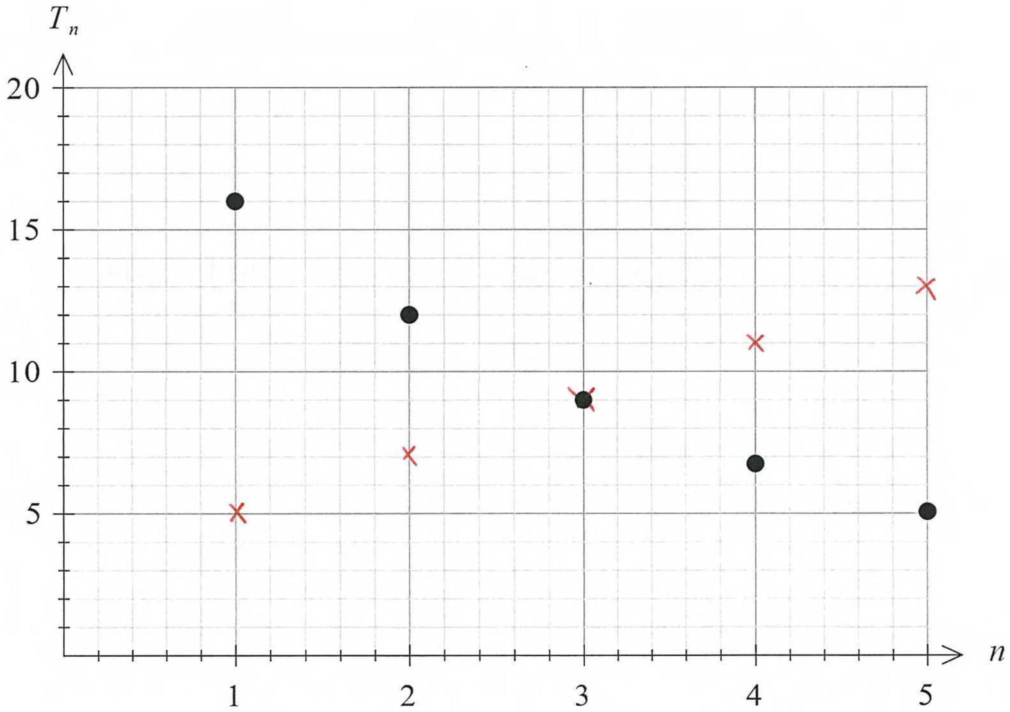
(c) As the value of n continues to increase, will the value of the term in the sequence increase, decrease or reach a steady state? [1]

increase

✓ correct answer.

2. (5 marks)

On the graph below a geometric sequence, T_n, has been plotted.



(a) For the geometric sequence, determine:

(i) The simplified ratio [1]

$\frac{3}{4}$ ✓ correct answer

(ii) The general rule [1]

$T_n = 16\left(\frac{3}{4}\right)^{n-1}$ ✓ correct answer

(b) Add the first five terms of the arithmetic sequence, $U_{n+1} = U_n + 2$, $U_1 = 5$ to the graph above. ✓ 3 correct terms [2]

✓ all correct terms

(c) Determine the values of n for which $U_n > T_n$. [1]

$n > 3$

✓ correct answer

3. (8 marks)

An arithmetic sequence has a fourth term of -3 and a tenth term of 51.

(a) Determine the rule for the n^{th} term of the sequence. [3]

$d = \frac{51-(-3)}{6}$ $T_1 = -30$ ✓ difference value

✓ first term

$= 9$ $T_n = -30 + 9(n-1)$

✓ general rule

(b) Calculate the 21[st] term of the sequence. [2]

$T_{21} = -30 + 9(21-1)$ ✓ substitution

$T_{21} = 150$ ✓ correct answer

(c) State the last term of the sequence which has a value less than 200. [3]

$200 > -30 + 9(n-1)$ ✓ substitution

$239 > 9n$ ✓ $n = 26$

$n = 26$ ✓ $T_{26} = 195$

$T_{26} = -30 + 9(26-1)$

$= 195$

4. (6 marks)

A wetland has a population of black-necked storks is initially at 20 and has a natural decrease of half the population per year. At the end of each year, 8 extra black-necked storks are introduced to the wetlands.

(a) Determine the recursive rule for the population of black-necked storks. [2]

$T_{n+1} = \frac{T_n}{2} + 8, \quad T_0 = 20$

✓ recursive rule

✓ $T_0 = 20$

(b) Determine the long-term steady state of the black-necked stork population. [2]

$S = \frac{8}{1-0.5}$ ✓ substitution

$= 16$ ✓ correct answer

(c) The wetlands would like to maintain a population of 24 black-necked storks. What is the yearly addition of black-neck storks required to produce this steady state given that all other conditions are to remain the same? [2]

$24 = \frac{x}{0.5}$ ✓ substitution

$x = 12$ ✓ correct answer

TRINITY COLLEGE

Mathematics Applications Unit 3/4
Test 2 2020

Section 2 Calculator Assumed
Sequences

STUDENT'S NAME ______________________________

DATE: Friday 22nd May **TIME:** 25 minutes **MARKS:** 27

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (8 marks)

A sequence is defined by the recurrence relation:

$T_{n+2} = a.T_{n+1} + b.T_n$ where $T_1 = x$ and $T_2 = y$.

(a) Determine the first five terms of the sequence if $a = -1$, $b = 1$, $x = 2$ and $y = 3$. [3]

$T_{n+2} = -T_{n+1} + T_n$ ✓ recursive rule

$T_1 = 2, \quad T_2 = 3$ ✓ T_1 + T_2

$T_3 = -1, \quad T_4 = 4, \quad T_5 = -5$ ✓ T_3, T_4, T_5

(b) The sequence 10, 20, 70, 200, 610 obeys the recurrence relation given. By using simultaneous equations or otherwise determine the values of a and b and hence state the recursive rule. [5]

$70 = 20a + 10b$

$200 = 70a + 20b$

$610 = 200a + 70b$

$a = 2, \quad b = 3$

$T_{n+2} = 2T_{n+1} + 3T_n$

$T_1 = 10, \quad T_2 = 20$

✓ One equation
✓ second equation
✓ a + b
✓ recursive rule
✓ T_1 + T_2

6. (7 marks)

A plantation has 4 800 trees. The plantation manager is interested in modelling what would happen if each year, 10% of the existing trees were cut down for timber and another 250 new trees are planted.

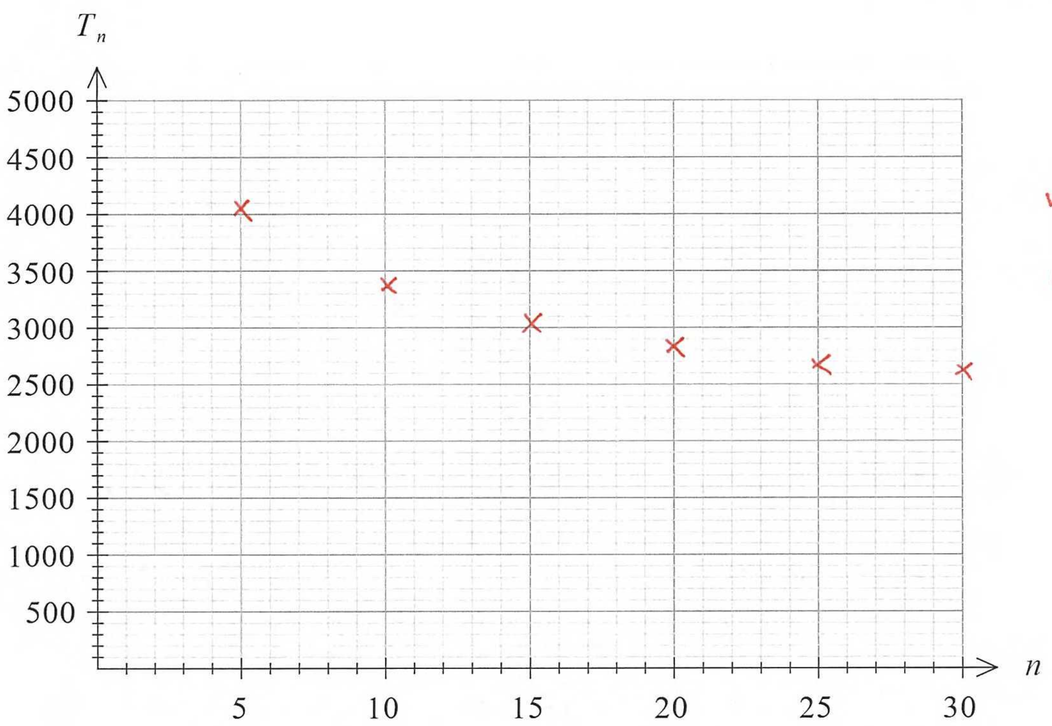
The number of trees, T_n, at the start of year n can be modelled by $T_{n+1} = 0.9T_n + 250$, $T_1 = 4800$.

(a) Use the recurrence relation to complete the missing values in the following table. [2]

n	5	10	15	20	25	30
T_n	4009	3391	3026	2811	2683	2608

✓ 2 correct
✓ All correct

(b) Plot the values from the table in part (a) on the axes below. [2]



✓ 3 correct
✓ All correct

(c) Comment on how the number of trees in the plantation is changing. [1]

Decreasing

✓ correct answer

(d) Does the model predict that eventually there will be no trees left in the plantation? Justify your answer algebraically. [2]

No, it will not reach 0.

$x = 0.9x + 250$

$x = 2500$

$x = \frac{250}{1-0.9}$

$x = 2500$

✓ no.
✓ shows steady state

7. (7 marks)

In ideal conditions a certain bacterium double their numbers every 15 minutes. A raw fish, under these conditions, with 180 harmful bacteria is left lying on a kitchen bench.

(a) Show that the ratio at which the bacteria increase every hour is 16. [1]

$r = 2^4$

$= 16$

✓ shows $2^4 = 16$

(b) State the recursive rule for the number of bacteria, where *n* is the number of hours the fish has been on the counter. [2]

$T_{n+1} = 16\,T_n, \quad T_0 = 180$

✓ recursive rule

✓ $T_0 = 180$

(c) Estimate the number of bacteria on the fish after 2 hours. [1]

$T_2 = 46\ 080$

After being left on the counter for 2 hours, the fish is cooked in an oven, where the bacteria are killed at a rate of 78% per hour. The fish is to be cooked for 3 hours

(d) Given that the fish should contain less than 500 bacteria to be considered safe to eat, will the bacteria reduce enough after being cooked for 3 hours for the fish to be eaten safely. Justify your answer. [3]

$T_{n+1} = 0.22\,T_n, \quad T_0 = 46080$

✓ shows new sequence

$T_3 = 490.66$

✓ calculates T_3

$\therefore$ fish can be consumed safely

✓ comments on fish consumption correctly.

8. (5 marks)

A sequence is defined by the rule $A_{n+1} = 2 - 3A_n$, where $n = 1, 2, 3, \ldots$ for $A_1 = k$. Given that k is a constant, determine k if $A_4 = -598$.

$A_2 = 2 - 3k$ ✓ subs A_2

$A_3 = 2 - 3(2 - 3k)$ ✓ subs A_3

$= -4 + 9k$ ✓ subs A_4

$A_4 = 2 - 3(-4 + 9k)$ ✓ subs $A_4 = -598$

$= 14 - 27k$ ✓ calculates $k = \frac{68}{3}$

$-598 = 14 - 27k$

$k = \frac{68}{3}$ or $22\,^2/_3$